Self-Excited Wire Method for the Control of Turbulent Mixing Layers

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A new method for the control of the evolution of a turbulent, two-stream (planar) mixing layer is reported. A (music) wire strung across the flow and placed in the zone near the trailing edge of the splitter plate is used to excite the flow. The wire is "self-excited" by the shear flow. The vibrating wire, whose frequency can be adjusted, transfers energy to the flow in a very efficient way due to a local high degree of coupling. Results are reported for a velocity ratio of 0.674 and $Re_{\theta}=1380$, with excitation frequencies between 85 and 400 Hz. Depending on the frequency of wire vibration, different modes of large structures are excited, determining the spreading rate of the mixing layer. It is also possible to control the directional evolution of the mixing layer and, thus, potentially the entrainment ratio between the two streams. This is achievable by adjusting the wire position in the lateral direction, inside the mixing layer. These characteristics are the result of the highly localized energy input of the wire into the shear layer, compared to other methods.

Nomenclature

f_e	= excitation frequency
Q	= integrated volume flux, $\int_{R_0}^{R_{0.9}} U(x, y) dy$
Q_0	= integrated volume flux at $X = 0.5$ mm
$(Q - Q_0)/Q_0$	= flow entrainment ratio
R	= velocity ratio, $(U_H - U_L)/(U_H + U_L)$
$R_{0.1}$	= velocity ratio truncated at $R = 0.1$
$R_{0.9}$	= velocity ratio truncated at $R = 0.9$
Re_{θ}	= Reynolds number based on initial momentum
	thickness
U	= local time mean velocity
U_L, U_H	= low-speed side, high-speed side velocities
u'	= turbulent fluctuation
x, y, t	= streamwise, lateral, time coordinates
•	= lateral coordinates truncated at $R = 0.5$
•	= nondimensionalized coordinate, $(y - y_{0.5})/\theta$
$\dot{ heta}$	= momentum thickness
y _{0.5} η θ	= nondimensionalized coordinate, $(y - y_{0.5})/6$

Introduction

ANY practical flow devices contain free shear flows, cold or reacting, in which large coherent structures are imbedded. In most of these devices mixing between different fluids, separate feeds or reactants and products, is of critical importance. Examples include ramjets, scramjets and turbojet combustors, certain laser systems, and many processes in the chemical and food processing industries. In the area of energy conversion, increasing requirements for the optimization of device volume and weight and tightening of environmental standards are pushing manufactures to the realization that active control of the processes taking place within a device will be necessary soon. One aspect is active control of the structure and the evolution of the flowfield inside a flow device. Such active control includes several disciplines: flow physics, flow actuation, practical actuator design, and implementation.

Although the mechanisms involved in active control of the planar mixing layer are relatively well understood at the present, the methods of control which include actuation methods, practical actuator design, and implementation are far from satisfactory. Usually, to achieve a desired increase in mixing rate either requires too much

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energy or is impractical because of the complexities in installing the actuator. Both problems negate the advantages of active flow control compared to passive control methods. Hence, there is an urgent need to develop new methods of active flow control.

In this paper a description is given of a fundamental type of actuator applied to a two-stream, turbulent, planar mixing layer and of the effects its implementation has on the structure and evolution of the flowfield. The study presented here was part of an effort to examine the developments of simple, low-energy consumption and "best-coupling" actuators for free shear flows.

The fundamental physics underlying the development of planar shear layers, natural and excited, have been the subject of much research during the last two decades. A wealth of information can be founded in the works of Ho and Huang¹ and Ho and Huerre.² Results for turbulent shear layers were reported by, among others, Wygnanski and Petersen³ and Fiedler and Mensing⁴ and were compiled lately by Dimotakis.⁵

Analysis of the locally excited, turbulent, planar mixing layer reported herein is based on concepts and measures advanced by previous researchers like Ho and Huang,¹ Ho and Huerre,² Dimotakis,⁵ and Wygnanski and Peterson.³ Examples are shear layer growth rate, forcing characteristics, receptivity, most amplified and subharmonic frequencies, vortex merging, and entrainment rate ratio.

After a brief description of the experimental facilities and techniques, the major findings are reported. A discussion of the governing mechanisms follows, and some recommendations for possible applications are given at the closure.

Facilities and Methodology

The study was performed in a small-scale wind tunnel, Fig. 1a. Mufflers were installed between the blowers of the two streams and the diffuser in order to dampen a periodic component in the flow caused by the blower vane passage frequency. The test section is connected to the two plenum chambers through a 16:1 contraction, and each stream exits through a cross section 63 mm (2.5 in.) high and 102 mm (4.0 in.) wide. Attainable velocities are 10–40 m/s on the high-speed side and 4–15 m/s on the low-speed side, with a turbulence level of 0.5%. The separation of the two streams is terminated in the test section using a splitter plate with a 3-deg sharp edge. The test section used was 300 mm (12 in.) long, made of plexiglass.

The planar shear layer was excited by a (music) wire strung across the test section through the two holes drilled in the side walls and placed in the region near the trailing edge, at several y coordinates, as shown in Fig. 1b. Several types of music wire (piano and guitar) were used with diameters in the range of 0.15–0.76 mm (0.006-0.03 in.). The wire length was 6–8 times the width of the test section to ensure

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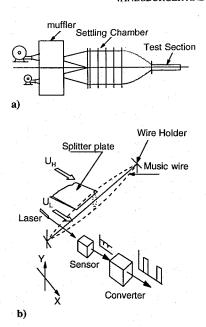


Fig. 1 Schematic view of a) wind tunnel and b) wire setup.

uniform spanwise excitation. The wire self-vibration frequency, 85–400 Hz in this study, was varied by changing the wire diameter (moment of inertia), material (modulus of elasticity), and tension. The amplitude of the wire vibrations in the test section was 6 mm for all cases.

The flowfield structure was surveyed using a hot-wire anemometer and some smoke-wire flow visualization (not presented here). The hot-wire probe (TSI type 1210 T1.5) was mounted on a traverse with three degrees of freedom. Data acquisition and probe location control were performed by a PC 486 computer via a DT 2801-A A/D board using direct memory access (DMA). Sampling rates of 27 kHz could be reached. Velocity signals collected were used to obtain maps of mean velocity and turbulent kinetic energy. Spectral analysis of the signals was done mostly on the personal computer using a dedicated Zonic PC2000 fast Fourier transform (FFT) board; during later experiments a Tektronix 2630 system was used. Graphical analysis and presentation of the data was done using Techplot and Image Pro-Plus on the PC-486 and AVS4 on a Silicon Graphics Workstation.

Two types of data were collected, time series and phase locked. The time-series data were used to obtain time-mean information of the flowfield. Naturally, these data wash out detailed (frequency specific) information about the shear flow. Phase-locked information was obtained with the help of a phase-reference (laser-detector) system, shown in Fig. 1b. The self-excited wire passed through the beam of an He-Ne laser which was targeted on a photodetector. The signal acquired from the detector provided an accurate measure of the excitation frequency and served as a trigger for the phase-locked sampling.

Results

Reported in this paper are the effects of local excitation on the evolution of a planar mixing layer with freestream velocities of $U_L=7.0\,\mathrm{m/s}$ on the low-speed side and $U_H=36\,\mathrm{m/s}$ on the high-speed side. This corresponds to a velocity ratio of $R=(U_H-U_L)/(U_H+U_L)=0.67$ and a momentum thickness Reynolds number of 1380, based on a momentum thickness of 3.0 mm at an axial distance 40 mm from the trailing edge of the splitter plate. The boundary layer on both sides of the splitter plate were intentionally kept turbulent. Both streams consisted of air and, thus, the density ratio was unity.

The characteristics of the natural, turbulent, shear flow were examined first. Autopower spectra were obtained at various stations in the axial direction near the outer boundaries of the shear layer. Very near the trailing edge the spectra constituted of broadband noise with no particular vortex shedding frequency discernible. At an axial location of 40 mm from the trailing edge a broad peak between 200–1000 Hz with a maximum about 550 Hz is observable

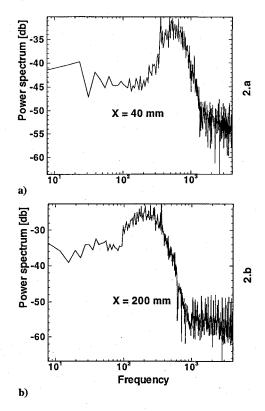


Fig. 2 Power spectra of the unexcited flow: a)x = 40 mm and b)x = 200 mm.

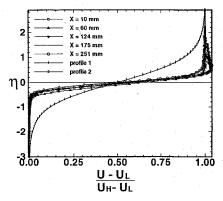


Fig. 3 Normalized velocity profiles across mixing layer.

(Fig. 2a). This station corresponds to an axial position of about one wavelength of the most unstable waves. Figure 2a also reveals that the turbulent case is characterized by a band of frequencies, unlike in the laminar case where a single most amplified frequency and its harmonics exist. This band moves to lower frequencies as the flow progresses downstream, as seen in Fig. 2b. Both of these characteristics were reported earlier by Wygnanski and Peterson³ and Wygnanski et al.⁶ The unexcited mean flow velocity profiles, for several axial locations, are plotted in Fig. 3 using similarity variables. The experimental data collapsed well. One should note that profile 2, given by the expression to follow, is a modification of the "tanh" profile shown as profile 1 and provides a better fit:

$$\frac{U - U_L}{U_H - U_L} = 0.5[1 + \tanh(4.0 * \eta)]$$
 $\eta = \frac{y - y_{0.5}}{\theta}$

Since the instability mechanism of the turbulent mixing layer is mainly controlled by the mean velocity profile of the flow,⁷ it will evolve in the far field according to its most unstable mode, despite the fact that many other modes can be excited in the near field. Thus, in the case of the turbulent mixing layer, the most amplified frequency and its subharmonics can still be determined from a stability analysis based on the mean velocity profile. The most amplified frequency calculated from profile 2 is 570 Hz, which is very close to the experimental result of 550 Hz.

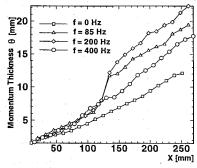


Fig. 4 Momentum thickness of mixing layer for different excitation frequencies.

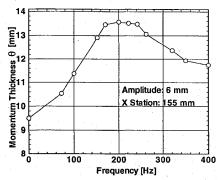
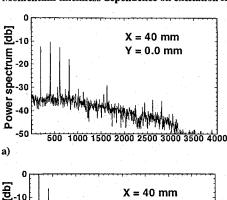
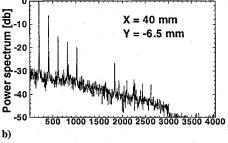


Fig. 5 Momentum thickness dependence on excitation frequency.





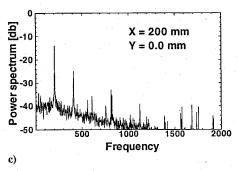


Fig. 6 Spectral analysis of mixing layer excited at 200 Hz.

For the work reported herein the excitation frequencies ranged from 85 to 400 Hz, which includes frequencies in a range from well below to frequencies near the most amplified frequency. The range of frequencies chosen was restricted by the physical properties of the available wires, the freestream velocities, and the difference

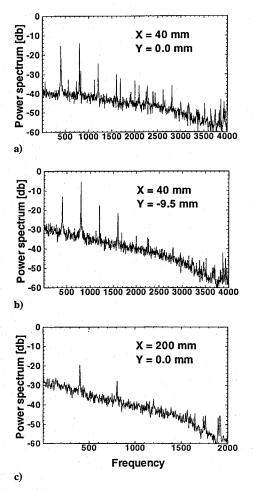


Fig. 7 Spectral analysis of mixing layer excited at 400 Hz.

between the velocities of the two stream. As an example, a 0.15-mm (0.006-in.) piano wire could only be excited with a freestream velocity above 10 m/s and a velocity ratio R>0.3. In all cases the wire had to be placed near the trailing edge in the mixing layer to be excited.

The effects of wire excitation are reported next, first time-mean data is presented followed by phase-locked information. The wire was placed close to the trailing edge of the splitter plate at several y coordinate locations; see Fig. 1b. The amplitude of vibration was always about 6 mm, of the same order as of the local mixing layer thickness. The effect of the wire excitation, using time-mean data, is first evaluated in terms of spatial evolution (growth) of the mixing layer. A commonly used measure for the extent of the mixing layer is its momentum thickness. Figure 4 shows the growth of the mixing layer momentum thickness as a function of downstream distance from the splitter plate, for unexcited flow and excitation at 85,200, and 400 Hz, with the wire positioned at y=0 mm and x=5 mm.

Several observations are made. The natural mixing layer spreads almost linearly. All excited cases exhibit increased growth. Excitation at frequencies equal or less than 200 Hz results in a significant shear layer spread, localized around $x=120 \, \mathrm{mm}$ (as will be shown later this corresponds to the location where vortices begin to merge), followed by a second linear region with the same growth rate as in the initial region.

The effect of excitation frequency on mixing layer growth has been examined and is shown in Fig. 5. It can be seen, indeed, that there is an optimum frequency around 160–240 Hz, in the subharmonics range of the most amplified frequency for the unexcited mixing layer.

The spectral evolution of the mixing layer is shown in Figs. 6 and 7 for excitation at 200 and 400 Hz, respectively. One can see that the flow strongly locked to the excitation frequency both in the near and far flowfields. Notice that for 200 Hz, at X = 40 mm and Y = 0 mm, the first harmonic contains more energy (Fig. 6a),

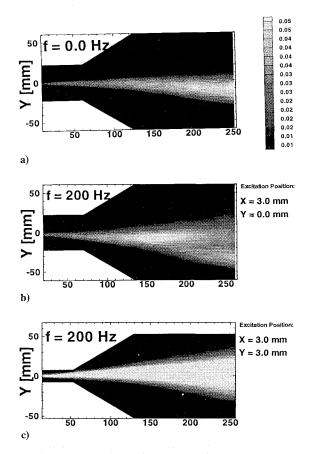


Fig. 8 Turbulent kinetic energy: a) unexcited, b) $f_e = 200$ Hz, symmetric wire, and c) $f_e = 400$ Hz, asymmetric wire.

whereas at y = -6.5 mm (Fig. 6b) the fundamental contains more energy. In contrast, in the case of 400 Hz (Figs. 7a and 7b) the first harmonic always has the highest energy. In both cases the far field is dominated by the excitation frequency (see Figs. 6c and 7c). It is the authors' view that Figs. 6a and 6b might indicate different coherent structures that coexist at the same location, a point that will be elaborated further in the following text.

Another way to examine the effect of excitation on the mixing layer development is to plot its boundaries as a function of axial distance downstream. Using a measure of turbulent kinetic energy $(u'/U)^2$, time-mean contour maps have been drawn and are shown in Fig. 8. Figure 8a is for the unexcited mixing layer; in Figs. 8b and 8c the wire was excited at 200 Hz and placed at a lateral coordinate of y = 0 and y = 3.0 mm, respectively. The data were acquired at the midplane location, in the transverse direction of the test section. It is clear from the images that the flow excitation by the wire increased the spreading of the mixing layer, the region of turbulent fluid, and, therefore, to some degree the region of mixed fluid, which is related to but not identical with the region displayed here.⁵ In both cases (Figs. 8a and 8b) the shear layer spreads more into the low-velocity side, consistent with existing knowledge.⁵ This situation can be changed by moving the exciting wire in the y direction. Figure 8c shows a case where the wire was placed at y = 3.0 mm and self-excited at 200 Hz. The mixing layer, in this case, spreads more symmetrically. The data also indicates a higher level of turbulence over a wider zone of the mixing layer. The asymmetric wire placement may, therefore, lead to an increased entrainment of fluid from the high-speed side into the mixing layer, compared with the symmetrical wire location of Fig. 8b. As a consequence, the proportions of fluid from the two streams contained in the mixing layer could be varied through small adjustments in the lateral location of the wire relative to the splitter plate, a point of importance to chemically reacting shear layers. The authors have to note that this possibility needs to be verified by scalar measurements.

After the simplicity and effectiveness of the self-excited wire method has been demonstrated, more information on the flowfield

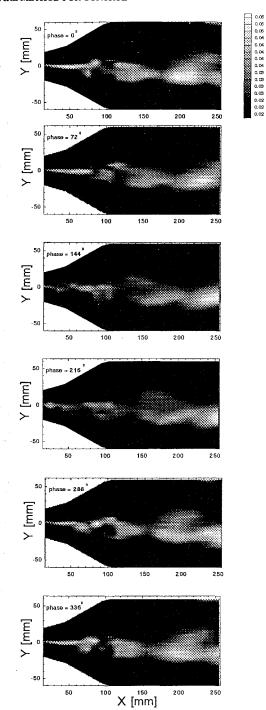


Fig. 9 Turbulent kinetic energy maps at six times during an excitation cycle for $f_e = 200$ Hz.

structure and evolution was sought. For that purpose phase-locked data was collected, analyzed, and presented in the x-y-t, space-time domain. Ensemble averages of phase-specific data for the mixing layer midplane were calculated. Turbulent kinetic energy maps (x-y)planes) for six instances during an excitation cycle, for 200-Hz excitation, are shown in Fig. 9. Examination of the near field of the flow reveals that the flow contains multiple coherent structures of different sizes. (A similar observation was made for excitation at 85 Hz but is not shown here.) One can see that at 0 phase angle, two large structures exist between the streamwise location of 50-100 mm. The structure on the high-speed side is ahead of the one on the low-speed side. These structures develop as the phase increases. Because of the differences in local phase speed, the structure on the low-speed side is retarded during the propagation, whereas the one on the highspeed side catches up and merges with the much larger structure ahead of it. This results in an increase of the local mixing layer thickness. The same maps are shown in Fig. 10 for an excitation frequency

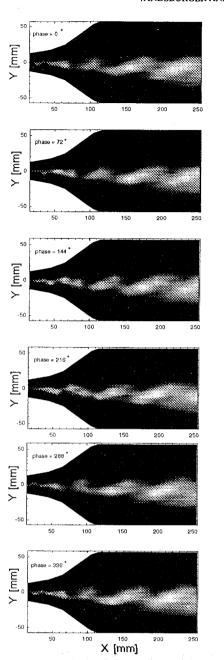


Fig. 10 Turbulent kinetic energy maps at six times during an excitation cycle for $f_e=400~{\rm Hz}$.

of 400 Hz. In this case, however, only a single large-scale structure is evident. No merging process is observed. This may be explained by the fact that the flow excitation occurs close the most amplified frequency. There seems, therefore, to be a fundamental difference in the response of the mixing layer to excitation below vs above the subharmonics (about 200 Hz) of the most amplified frequency.

Enhancement of freestream fluid entrainment into the mixing layer is an important measure to evaluate the improvement of mixing. Figure 11 shows comparisons of flow entrainment under different conditions. As can be seen, the 200-Hz excitations (case a, y=0 mm and case b, y=3 mm), indeed, have the highest flow entrainment ratio, which for case a is close to twice the unexcited case at the streamwise location of 250 mm. As expected from results reported earlier, 400-Hz excitation has a lesser effect on the entrainment ratio, 7.0 as compared to the over 8.0 for the case of 200-Hz excitation. Also, one notices that the adjustment of wire location along the lateral directions does have some effect on the overall flow entrainment, for instance, at x=250 mm, the flow entrainment ratio for case a is 9.0 whereas for case b it is 8.0.

The effect of the excitation on the flowfield structure in terms of propagation of coherent structures was examined using three-

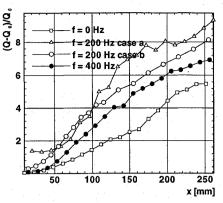


Fig. 11 Flow entrainment ratio under different excitation frequencies; case a: excitation position at x = 3 mm, y = 0 mm and case b: x = 3 mm, y = 3 mm.

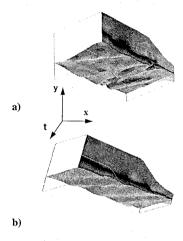


Fig. 12 Phase-mean isovelocity surface, $(U-U_L)/(U_H-U_L)=0.5$ in the x-y-t space: a) $f_e=200$ Hz and b) $f_e=400$ Hz.

dimensional views of phase-mean velocity surfaces. Shown in Figs. 12a and 12b are isovalue surfaces of $(u-U_L)/(U_H-U_L)=0.5$ for 200- and 400-Hz excitations, respectively. Clearly, the 200-Hz excitation exhibits a stronger effect on the flowfield structure. This is most noticeable in the near field where the "zigzag" shaped waveform is an indication of interaction between multiple structures.

Discussion

The self-excited wire method has proven very simple, rather effective, and does not require external energy input. Although a full dynamic analysis of the mechanism is still underway, a heuristic explanation will be given next. It has been observed that the wire will not vibrate in either freestream of the mixing layer. It was also observed that the path of the wire is elliptical in the x-y plane, with the major axis in the y direction, typically 6 mm long, and the minor axis in the x direction, with axis ratio 2–3:1. After an initial transient, the wire will always follow an elliptical path which moves it from the low-speed side to the high-speed side of the shear layer and back (near the trailing edge of the splitter plate). The amplitude of the wire (self-) vibration in the cross stream (y) direction can be controlled and, in order to be effective, should be of the same order as of the local mixing layer thickness. Therefore, the wire is traveling between the low- and the high-speed streams experiencing a time varying drag force, which is counteracted by the elastic (spring) reaction of the wire. The balance of these forces results in a limit cycle for the wire trajectory.

Despite the fact that the excitation amplitude was kept constant, best results were achieved for the subharmonic excitation. Examination of the space-time, x-y-t domain revealed the existence of multiple coherent structures in the near field for subharmonic excitation but not for higher excitation frequencies. In contrast, in the far field the flow always locked on the excitation frequency, i.e., far from the excitation point the response frequency of the flow always equaled the excitation frequency. This demonstrates

the strong influence the music wire excitation can exert upon the

Even though the near-field spectra of the unexcited mixing layer show a peak around 550 Hz, the most effective excitation frequency was found to be around 200 Hz. This is no contradiction, since the effectiveness is judged here in terms of mixing layer spread. As was reported already, for excitation frequencies of 200 Hz or lower the near field contained multiple structures, which paired while traveling downstream. This pairing leads to a dramatic increase in mixing layer thickness, at x=120 mm, see Fig. 5. This pairing-like event can be seen in Fig. 9, occurring somewhere between the axial position of 100 and 150 mm. The spectra of the 200-Hz excitation case, shown in Fig. 7, also supports this picture.

The response of the mixing layer to the wire excitation demonstrates, once again, the importance of efficient coupling of the perturbation energy into the flow. The coupling efficiency depends on three parameters: location of perturbation, frequency, and amplitude of the perturbation. The wire was placed near the trailing edge of the splitter plate due to the known high receptivity of the shear layer in this region. The frequency of excitation should be in the subharmonics range of the natural flow, although with large enough amplitude the flow will lock to almost any frequency. Whether the flow perturbation caused by the actuator can be considered small depends on the amplitude and location of excitation. In contrast to vibrating plate arrangements, which perturb the whole boundary layer leaving the splitter plate, the vibrating wire does not interfere with the incoming flow. As such, this type of excitation is more local compared to other methods reported in the past. The only requirement related to the excitation amplitude, using the wire, is that it be of the same order as the local shear laver thickness (near the trailing edge). This requirement is set by the physical characteristics of the wire, i.e., for it to reach a self-sustaining limit cycle, and not by minimum energy required to excite the

Before discussing some potential applications, one has to note that in the work reported here the wire vibrated in its fundamental mode, resulting in uniform excitation along the splitter plate. As a consequence the excited structures are two dimensional. Additional improvement in the mixing between the two fluids can be achieved through three-dimensional mixing layer evolution. Using a vibrating wire this can be realized by causing it to vibrate in higher modes (which was demonstrated in the authors' laboratory).

Potential applications are proposed in two areas. In the area of aerodynamics a vibrating (self-excited) wire can be placed in front of the airfoils to change their stall characteristics by decreasing the size of the separation bubble. Variation in the wire frequency of vibration can be achieved mechanically or thermally. Such an application has been demonstrated (unpublished) for the shortening of a separation bubble formed by a forward facing step by about two-thirds. One should, though, keep in mind the drag on the wire when considering airborne applications. An application even more natural is in the area of the mixing of two separate fluids. This could be particularly useful in reacting flows, like many step stabilized combustors, although it has to be noted that the effectiveness of the wire method in flows with heat release has yet to be determined. When compared with alternatives such as acoustic bulk modulation, thermal modulation, and mechanical trailing-edge actuation, the self-excited wire method is attractive due to its simplicity and zero energy input requirement.

Summary

Perturbation and control of the evolution of a two-stream turbulent, planar, mixing layer was achieved through the use of a flowexcited wire mounted in front of the trailing edge of the splitter plate. Excitation of the mixing layer at frequencies between 85 and 400 Hz was attained. In all cases the wire vibration amplitude was about 6 mm. For all excitation frequencies, the response frequency of the flow in the far field was equal to the excitation frequency. i.e., complete locking of the flow was observed. In the near field the response of the flow depended on the excitation frequency. For subharmonic excitation frequencies multiple structures were observed, whereas for frequencies excited close to the fundamental only single structures existed. The effect of the excitation was to increase the mixing layer momentum thickness in all cases. However, it was found that the greatest increase, greater than a factor of two. was attained for frequencies around 200 Hz. This marked increase in momentum thickness is attributed to interaction (pairing) between several structures in the near field.

Using the wire excitation it was also possible to control the directional evolution of the mixing layer. Unexcited and symmetrically (with respect to the splitter plate) excited mixing layers spread more into the low-speed side (bottom flow). By displacing the wire slightly in the vertical direction off the symmetrical location, this situation could be arbitrarily altered. Small displacements in the wire (actuator) location offer, therefore, a potential method for control over the entrainment ratio of the two fluids; of crucial importance to mixing and reacting flows.

In conclusion, the vibrating wire actuator offers an interesting alternative for shear flows actuation. Its virtues, compared with the standard techniques are 1) being self-excited, requiring no external energy input, 2) coupling into the flowfield locally, thus not creating a bulk disturbance, and 3) perhaps enabling precise control over the mixing ratio between the two streams.

Acknowledgments

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